

# Electrical Circuit I

## *Final Exam*

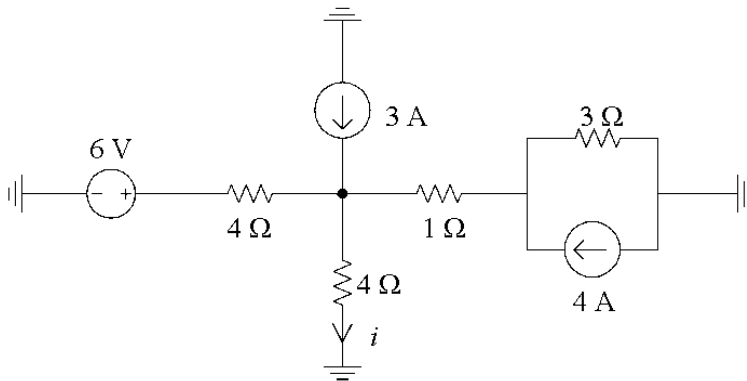
*28 June 2017*

Name & S.N: \_\_\_\_\_

- There are 12 pages in this final, including this cover page. Please check that you have them all.
- Please write your name in the space provided above, and circle the name of your recitation instructor along with the time of your recitation.
- **IMPORTANT:** The problems in this exam vary in difficulty; moreover, questions of different levels of difficulty are distributed throughout the exam. If you find yourself spending a long time on a question, consider moving on to later problems in the exam, and then working on the challenging problems after you have finished all of the easier ones.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. When finished, enter your answer to each question in the corresponding answer box that follows the question.
- Remember to include the sign and units for all numerical answers.
- You have 3 hours to complete this final.

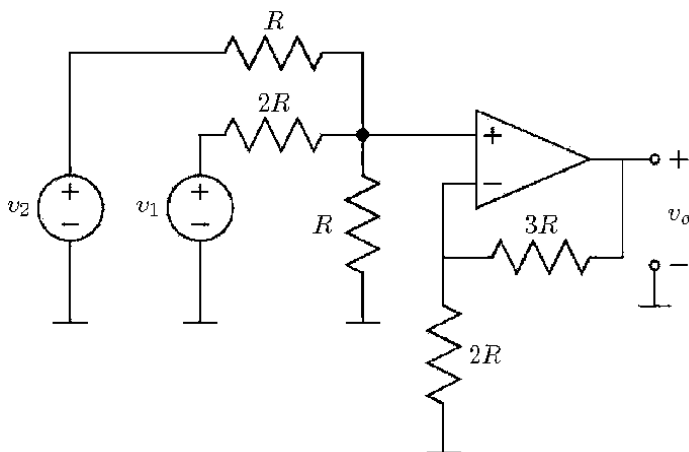
## Part A: Resistive Circuit Analysis

**Problem #1\_A:** Determine the current ( $i$ ) in the network below.



$i =$

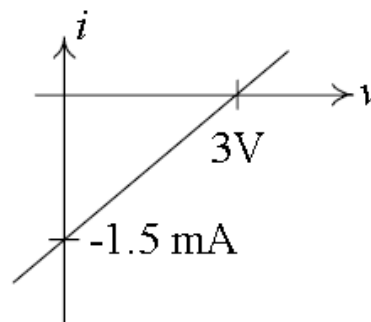
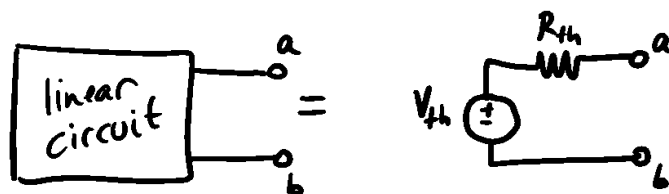
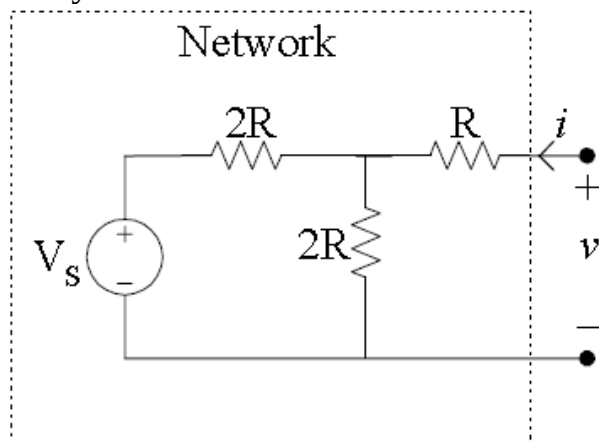
**Problem #2\_A:** Derive an expression for  $v_o(t)$  in terms of  $v_1(t)$  and  $v_2(t)$ :



$V_o(t) =$

## Part B: Thevenin/Norton Equivalent Circuits

**Problem #1\_B:** This problem involves a network (N) that is implemented with three resistors and a voltage source as shown below. Its terminal characteristics are also given graphically below:



- From the graphical data given above, determine numerical values for the parameters of the Thevenin equivalent of the network.

$R_{TH} =$

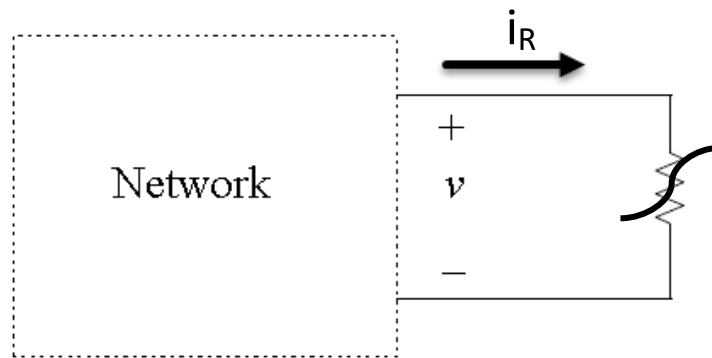
$V_{TH} =$

- Determine numerical values for the parameters  $V_S$  and  $R$  that characterize the implementation of the network shown above (N).

$R =$

$V_S =$

- The same network (N) is connected to an external nonlinear resistor with the voltage-current relation of  $v = k i_R^2$  as shown below. Determine the value range of  $k$  so that the connection does not lead to any possible solutions for  $(i_R)$  or  $(v)$ ?



$k$

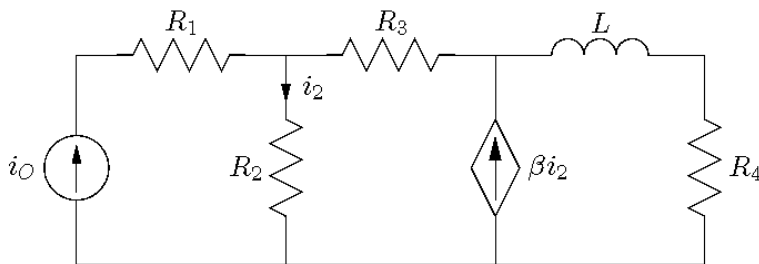
- A load resistor is connected across the terminals of the same network (N). Determine the value of load resistance that will result in the maximum power dissipated in the load resistor. Also determine the maximized load power.

Optimum Load Resistance:

Load Power:

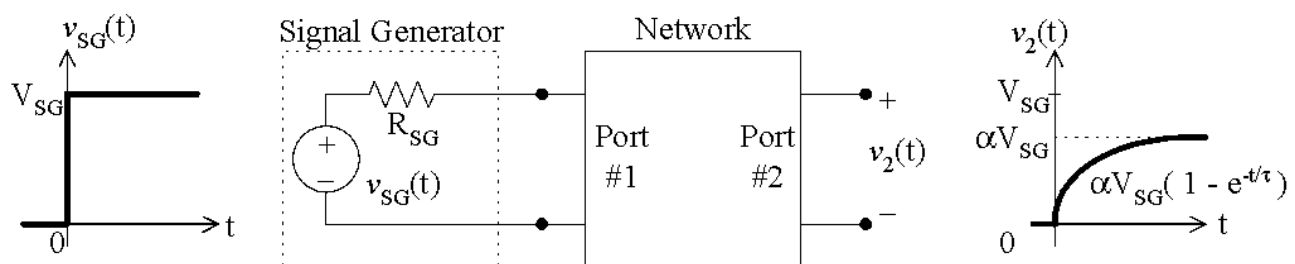
## Part C: First Order Circuits

**Problem #1\_C:** What is the time constant  $\tau$  for the circuit depicted in Figure below?

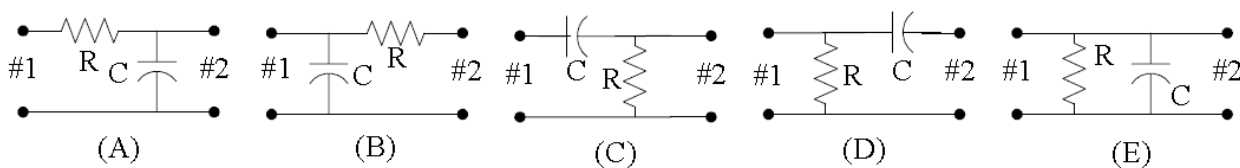


$\tau =$

**Problem #2\_C:** A signal generator having Thevenin resistance  $R_{SG}$  is connected to Port #1 of a two-port network as shown below. At  $t = 0$ , the Thevenin voltage  $v_{SG}(t)$  of the signal generator takes a step from zero to  $V_{SG}$ , and the voltage  $v_2(t)$  is measured at Port #2 as shown below with the port open-circuited. Note that  $\alpha$  is a unitless constant satisfying  $0 < \alpha < 1$ , and  $\tau$  is a time constant. Assume that the Thevenin voltage of the signal generator is zero for a very long time prior to the step

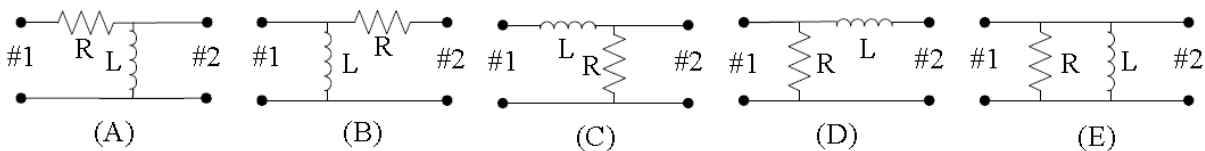


- Which of the following could be the two-port network? (Express your reasons)



Network (Circle One):            A            B            C            D            E

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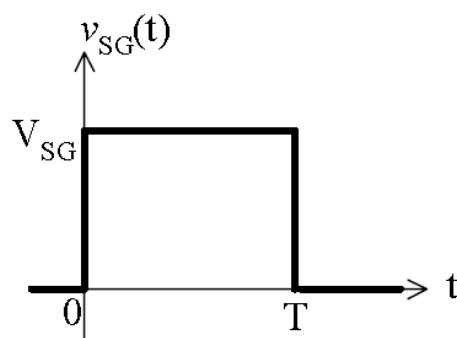
Network (Circle One):            A            B            C            D            E

- Determine the values of  $R$  and  $L$  in the network you chose in the last part. Express the values in terms of  $V_{SG}$ ,  $R_{SG}$ ,  $\alpha$  and  $\tau$ :

R=

L=

- The Thevenin voltage  $v_{SG}(t)$  of the signal generator now produces the pulse having amplitude  $V_{SG}$  and duration  $T$  shown below. Determine the voltage  $v_2(t)$  measured at Port #2 for  $t \geq 0$  with the port open-circuited. Express  $v_2(t)$  in terms of  $V_{SG}$ ,  $R_{SG}$ ,  $T$ ,  $\tau$  and  $\alpha$ . Assume that the Thevenin voltage of the signal generator is zero for a very long time prior to the pulse.



$V_o(t)=$

## Part D: Second Order Circuits

**Problem #1\_D:** A car travels at a constant speed along a bumpy road. As a result of the bumps, the velocity of the car's axle perpendicular to the road is  $V(t)$ . Taking into account the dynamics of the car's suspension system, the equation of motion for the body of the car (the chassis) can be formulated as follows:

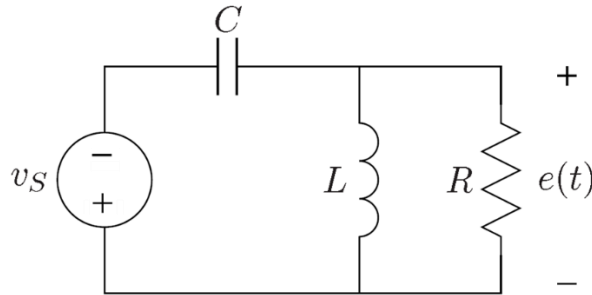
$$M \frac{du}{dt} + Bu(t) + K \int_{-\infty}^t u dt = -M \frac{dV}{dt} \quad (1)$$



where  $u(t)$  is the velocity of the body of the car (relative to the axle),  $M$  is the mass of the car,  $K$  is the spring constant of the springs connecting the body to the axle, and  $B$  is the coefficient of viscous damping for the shock absorbers.

In this problem, you will be asked to make a circuit model corresponding to the equation of mechanical dynamics from Equation 1, and then you will use the model to investigate the car's motion due to the bumpy road.

**Note:** Each of the parts of this problem can be worked independently of the others.



- The above-mentioned circuit is proposed to model Equation 1, where the node voltage  $e(t)$  is identified with the car body's velocity  $u(t)$ . Determine the values of  $v_s$ ,  $C$ ,  $L$ , and  $R$  in terms of the parameters in Equation 1 so that the solution for  $e(t)$  will be identical to the solution of Equation 1 for  $u(t)$ .

$V_S =$

$C =$

$L =$

$R =$

- Referring to the circuit model, assume that the response is oscillatory.
  - What is the frequency of oscillation  $\omega_{osc}$ ?
  - How long must one wait for the amplitude of the transient oscillation to decay to  $1/e$  of its initial amplitude?

$\omega_{osc} =$

$t_{1/e} =$

- Now assume that it is desirable to avoid oscillatory transient behavior. What is the maximum value of  $R$  in this case?

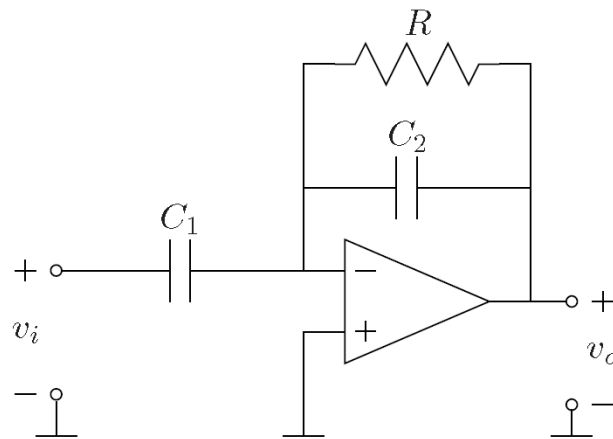
$R_{max} =$

## Part E: Sinusoidal Steady State Analysis

**Problem #1\_E:** Q. Rius is an engineer at Inlet Inc. Tasked with building a filter, Rius comes up with the op-amp circuit shown below, and applies the input signal  $v_i = A \sin \omega t$ . The output  $v_o(t)$  is of the form

$$v_o(t) = V_o \sin(\omega t + \theta)$$

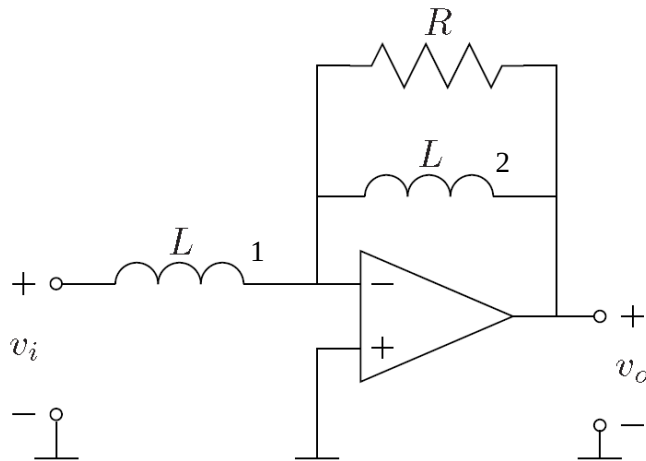
Determine the value of  $V_o$  and  $\theta$  in terms of  $A$ ,  $R$ ,  $C_1$ , and  $C_2$ .



$$v_o(t) =$$

**Problem #2\_E:** Let the network depicted in Figure below has certain filtering characteristics.

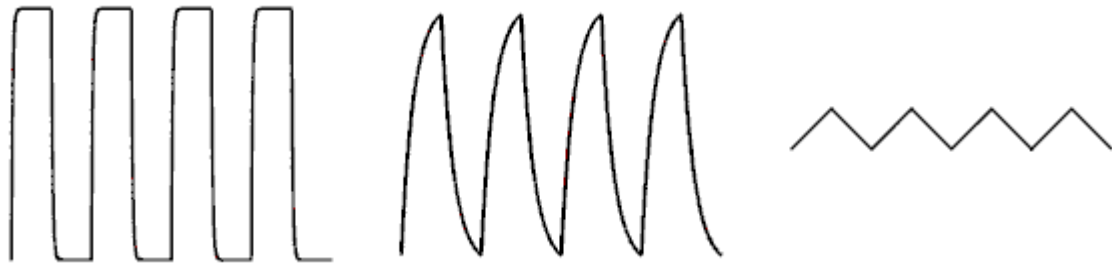
- Determine the frequency response (Transfer function) of the circuit:



$$H(j\omega) = \frac{v_o}{v_i} =$$

- Which of the Low-pass or High-pass responses could be achieved through the network? (Express your reasons)

- Which of the following waveforms could be obtained at the output stage when a square wave signal passes through the filter. (Express your guess and the corresponding reasons)



Good Luck  
Ali Abdolali  
&  
Teacher Assistants